In section 2.1, the author compared the F-ratios of having technical replicates and without. The ratio of having two technical replicates is larger because the impact of the measurement error \sigma^2 has shown to be reduced. In addition, due to the non-orthogonal block structure in the phase 1 design, the structure of the ANOVA table is different when fitting treatment first or block first. The variance components of the ANOVA table can be estimates using REML or a simple linear combination. Then it is possible to do a weighted analysis of the treatment estimates.

Weighted analysis is performed when the design is non-orthogonal block structure. Hence, treatment effects may be spread across different strata. Weighted analysis is to estimate the treatment across different strata; another term is called inter-block analysis.

In section 2.2, it shows the order of fitting can affect the test for the treatment effects. More specifically, if the plant is fitted first, there is a presence of array effects when applying the test. However, if the array is fitting first, the array effects are then sweep out when the test for the treatment effects is applied. However, the coefficients of the intrablock variance component, i.e. \sigma^2, have increased from 1/2 to 2/3 for the analysis of fitting plant first to fitting the array first.

Section 3 introduced a simpler experiment with 2 treatments, 4 biological replicates, i.e. plants, and 2 technical replicates. The aim is trying to compare the multiple dye swap design and alternating loop design. Two designs are isomorphic with respect to treatments, because two different treatments are allocated to every array for both designs, and the number of treatment assigned to each dye are also the same. However, two designs are not isomorphic with respect to plants, because in the multiple dye-swap design has four sets of two arrays where each set contains both technical replicates of each plant. The alternating loop design has 2 sets of four arrays where each set contains only one of the two technical replicates of each plant.

REML method is applied here to derived the inversed of the fisher information matrix, this matrix is use to estimate the variance components using the fisher’s scoring algorithm (Thompson and Patterson).

The variance of the mean squares can also be derived from the inversed of the fisher’s information matrix. The estimated variance of \sigma^2 + 2\sigma\_\pi^2 = \theta\_2 + \theta\_3 is given by the sum of the four elements in the bottom right 2-by-2 corner of the inverse, because these four elements are associated with \sigma^2 + 2\sigma\_\pi^2 = \theta\_2 + \theta\_3.

The mean squares are distributed as is chi-square. To estimate the variances of the mean square requires the linear combination of mean squares (Crump, 1946)

Approximated chi-square distribution is based on the approximation formula described by Satterthwaite (1946). The approximated estimates from the observed mean squares are thought to be similar to the exact chi-square distribution.

The moments are mathematical definition of the distribution of a random variable. There are two types of two moments: moments about the origin, i.e. E(X^t), and moments about the mean,   
E[(x - \mu)^t].

The first two moments can be computed from the Gamma distribution. This shows the first moment is the mean of the distribution, and second moment is the variance of the distribution.

The first moment of the distribution of the random variable X is the expectation operator, i.e., the population mean. The second moment is wildly used and measured the “width” of a set of points, i.e. the variance.

The effective degrees of freedom (EDF) are determined by twice the square of mean divided by the variance.

How well am I estimated the mean squares? We can study it by computing its variances.

DF tell how well the estimating particular the parameter

More DF the better

MDSD the array is orthogonal to the plant

ALD the array is not orthogonal to the plant. The author suggested the use of singular value decomposition to define the contrasts.

Different set of contrasts of plants contain different amount of array information, i.e. efficiency factors. This is completely different to the balance-incomplete block design, where every contrast contains same amount of the treatment information.